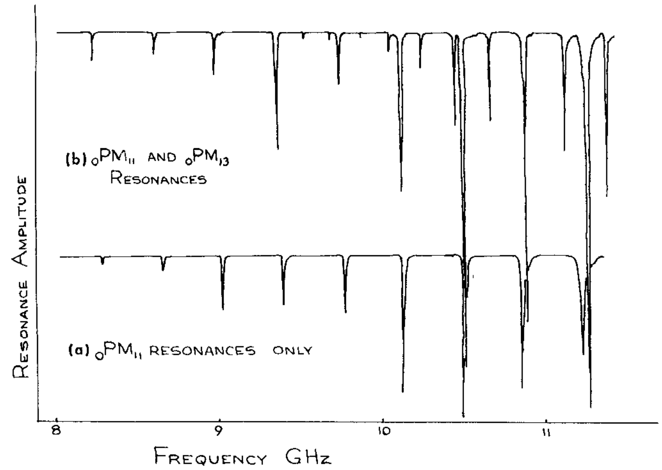

Fig. 4. Resonance spectra in (a) channelled and (b) unchannelled guides.

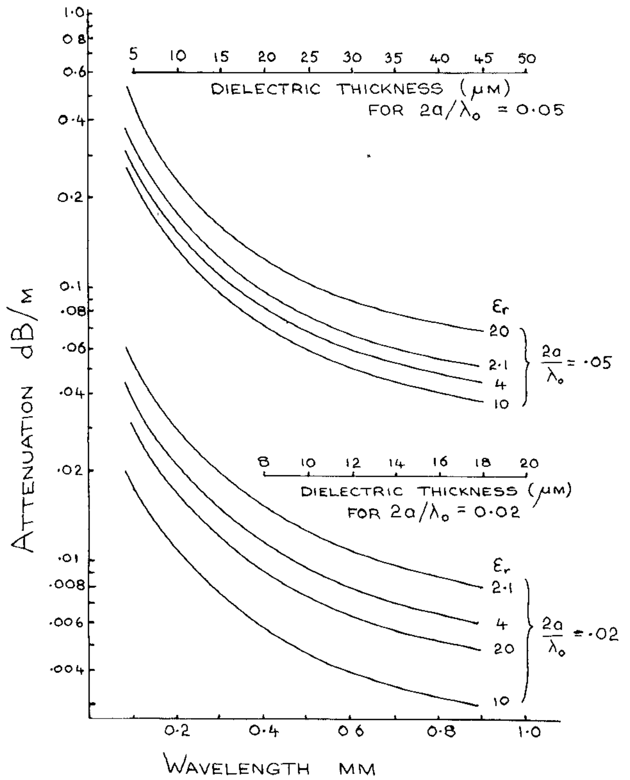

Fig. 5. Variation of dielectric attenuation with thickness and permittivity.

mittivity results in a large part of the electromagnetic energy traveling outside the dielectric and, therefore, not contributing to the dielectric loss.

Radiation loss from the open H-guide structure has been shown to be significant in certain circumstances [6]. It can, however, be reduced to a low value by increasing the height of the guide walls.

VI. Conclusions

Modified H-guide, in which a thin dielectric film is supported across the guide by channels in the conducting planes, appears potentially to offer advantages as a guiding structure for short millimeter and submillimeter wavelengths. The channel may be effective in suppressing higher order modes and allowing wider plane separation and, therefore, lower loss. Possible forms of construction are shown in Fig. 1(c) and (d). The use of a higher permittivity thin film of dielectric may reduce the loss still further. H-guide structures with channels have been investigated experimentally at 3-cm wavelength and the performance outlined verified. Guides are now being constructed for 0.3-mm wavelength operation for investigation using our HCN laser.

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A Simplified Circuit Model for Microstrip

HERBERT J. CARLIN

The advantage of a network model for a physical structure is that the model, if correctly established, implicitly contains the physical constraints of the actual system, and these constraints need not subsequently be called into play for every new case. A recent example is the use of coupled lines [1] to model longitudinally uniform but transversely inhomogeneous waveguides. The network model for a cylindrical waveguide loaded concentrically with a dielectric rod comprised a TE and a TM transmission line coupled together, and the properties of this model demonstrated that, surprisingly, the smooth lossless waveguide structure could support complex eigenvalues as well as backward waves. The general network idea stems from Schelkunoff [2] who established that uniform metallic-bound lossless guide structures can be represented by an infinite number of coupled TE and TM transmission lines. The practical approximating network model is obtained by appropriately truncating the infinite Shelkunoff representation [1].

In this short paper we show how a pair of coupled lines can give an extremely simple model for microstrip dispersion. We take a TEM transmission line and a TE line and form a distributed circuit with these two lines coupled together. The uncoupled lines propagate the ordinary TEM and TE modes. The coupled circuit automatically represents a pair of modes which are no longer TEM or TE, but instead are the two lowest order hybrid modes that exist on the stripline. In effect, circuit theory does the work in producing the required modes.

The pair of coupled lines modeling the microstrip is shown in Fig. 1. The circuit model for the physical structure is based on the fact that TEM- and TE-type modes excite each other by virtue of the presence of the dielectric substrate. It is also assumed in the model that the uncoupled TEM and TE modes propagate at the same velocity at very high frequencies, i.e., there is a common value of $\bar{\epsilon}$ for both of the lines.

The series-impedance and shunt-admittance matrices *per unit length* for the pair of coupled lines in Fig. 1 are

$$Z = p\mu_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad Y = p\epsilon_0 \begin{bmatrix} \bar{\epsilon} & c_{12} \\ c_{12} & \bar{\epsilon} \end{bmatrix} + \frac{1}{\mu_0 p} \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{K}^2 \end{bmatrix}. \tag{1}$$

Here $p = \sigma + j\omega$ is complex frequency, and ϵ_0, μ_0 are the constitutive constants of free space. There are therefore only three constants used for the simple circuit model: $\bar{\epsilon}$, the *effective* static dielectric constant; $\mathcal{K}$, the cutoff wavenumber for the uncoupled TE mode; and the coupling capacitance $c_{12} = k\bar{\epsilon}$, where $0 \leq k \leq 1$ is the capacitive coefficient of coupling. The effective dc dielectric constant is given by the static relation

$$\bar{\epsilon} = \left(\frac{z_0}{\bar{z}_0}\right)^2 \tag{2}$$

Manuscript received February 20, 1973; revised April 23, 1973. This work was supported in part by NSF under Grant GK-31012X.
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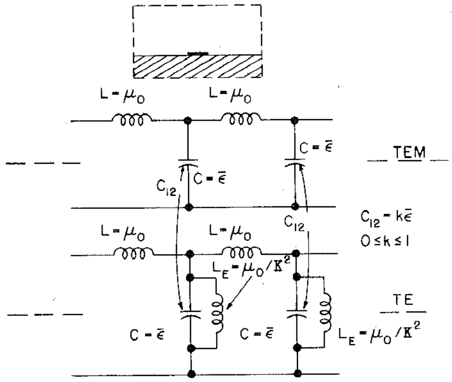

Fig. 1. TEM–TE coupled-line model per unit length for microstrip.

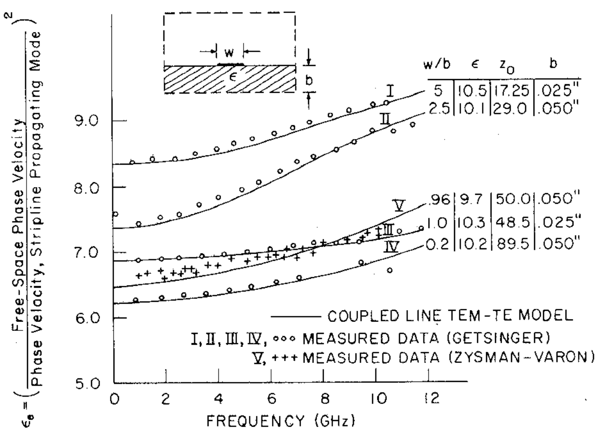

Fig. 2. Coupled line versus experimental dispersion curves for microstrip.

where the characteristic impedances of the microstrip are z_0 with air dielectric, and $\bar{z}_0$ with substrate relative dielectric constant ϵ_s. The values of $\bar{z}_0$ and $\bar{\epsilon} \equiv \epsilon_e(0)$ in the examples that follow are taken from Getsinger's data [4] which in turn are based on the program MSTRIP [3]. However, Wheeler's equations for characteristic impedance [6] give nearly the same fit, though somewhat poorer, to experimental dispersion data. Thus for the microstrip dimensions listed on curves I, II, III, and IV, in Fig. 2, we have the following:

	Getsinger		Wheeler[a]	
Fig. 1	$\bar{z}_0$	$\epsilon_e(0)$	$\bar{z}_0$	$\epsilon_e(0)$
I	17.25	8.36	17.01	8.51
II	29.0	7.38	28.4	7.64
III	48.5	6.88	48.3	6.87
IV	89.5	6.24	88.7	6.24

[a] Calculated using P. Penfield's program MARTHA.

The squared eigenvalues γ^2 associated with ZY are given in normalized form as

$$-\frac{\gamma^2}{\mathcal{K}^2} = \Omega^2 - \tfrac{1}{2} \pm \sqrt{k^2\Omega^4 + \tfrac{1}{4}}. \tag{3}$$

The normalized angular frequency is

$$\Omega = \frac{\omega}{\omega_c}, \qquad \omega_c = \frac{\mathcal{K}v_0}{\sqrt{\bar{\epsilon}}} \tag{4}$$

where ω_c is the cutoff frequency of the uncoupled TE line, and the free-space velocity is

$$v_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}}.$$

The relative effective dielectric constant is

$$\epsilon_e(\omega) = \left(\frac{v_0}{v(\omega)}\right)^2, \qquad v^2(\omega) = -\frac{\omega^2}{\gamma^2} \tag{5}$$

where $v(\omega)$ is the phase velocity of propagation in the microstrip corresponding to the plus sign in the dispersion relation (3). This mode propagates down to dc. The other mode [minus sign in (3)] is cut off at low frequencies. Then from (3), (4), and (5)

$$\epsilon_e = -\left(\frac{\gamma^2}{\omega^2}\right)\cdot v_0^2 = \bar{\epsilon} - \frac{\mathcal{K}^2v_0^2}{2\omega^2} + \sqrt{(k\bar{\epsilon})^2 + \left(\frac{\mathcal{K}^2v_0^2}{2\omega^2}\right)^2}. \tag{6a}$$

Alternatively, if we "derationalize" (6a)

$$\epsilon_e = \bar{\epsilon} + \frac{2(k\bar{\epsilon}\omega)^2}{\mathcal{K}^2v_0^2 + \sqrt{(2k\bar{\epsilon}\omega^2)^2 + (\mathcal{K}^2v_0^2)^2}} \tag{6b}$$

which is probably more convenient for calculations, especially when ω is small. From (6b) we verify that $\epsilon_e(0) = \bar{\epsilon}$.

The second constant, the coefficient of coupling k, is easily found from the following reasoning. When the microstrip is excited, the fields outside the dielectric decay to a negligible level within a wavelength λ of the dielectric interface. Thus, as $\omega \rightarrow \infty$, $\lambda \rightarrow 0$, the fields may be considered as entirely confined within the substrate and we may assume $\epsilon_e(\infty) = \epsilon_s$, the dielectric constant of the substrate material. This should be true for all modes. However, because of the simple form of the model, we can only impose the infinite frequency condition on the fundamental mode [plus sign in (3)]. Using this constraint in (6a) gives

$$k = \frac{\epsilon_s - \bar{\epsilon}}{\bar{\epsilon}}. \tag{7}$$

There is only one more parameter to be determined, the TE cutoff wavenumber $\mathcal{K}$. This is found by equating the frequency for the point of inflection ω_i calculated from (6) to the value given by Getsinger [4]. From (6) we set

$$\frac{d^2\epsilon_e}{d\omega^2} = 0$$

which yields

$$\frac{\omega_i^2}{\omega_c^2} = \frac{R}{k}, \qquad R = \frac{(2\sqrt{7} - 1)^{1/2}}{6}. \tag{8}$$

The Getsinger [4] equation gives

$$\omega_i = \frac{\bar{z}_0}{2\mu_0 b\sqrt{3G}}$$

where b is the substrate thickness, and G is a semi-empirical parameter that depends on $\bar{z}_0$. In our circuit model for best average fit we use the relation for G (differing from Getsinger [4])

$$G = 0.500 + 0.001\bar{z}_0^{3/2}. \tag{9}$$

Then from (8)

$$\mathcal{K}^2 = \frac{k}{R}\,\frac{(2\pi)^2}{12Gb^2}\,\bar{\epsilon}\left(\frac{\bar{z}_0}{376.7}\right)^2. \tag{10}$$

We now merely calculate the three parameters of (2), (7), and (10), all determined solely from the microstrip physical dimensions and the dc substrate dielectric constant. Our simple model (Fig. 1) and dispersion relations (6a), (6b) are then completely specified. No recourse to interpolation to specific experimental dispersion data is required.

Fig. 2 shows dispersion curves calculated from our simple two-coupled line model and compared with experimental data [4], [5]. The simple network model fits the Getsinger data very well. The same model also fits the measured dispersion data of Zysman and Varon [5], as shown.

Fig. 3 shows dispersion curves calculated from the model for the mode which propagates to dc compared with the mode which is cutoff below a finite frequency. Note that as we would expect, the mode which is cutoff at dc exhibits a higher and higher cutoff frequency as its paired propagating mode becomes less dispersive in character. By setting (3) to zero and using the minus sign, we obtain

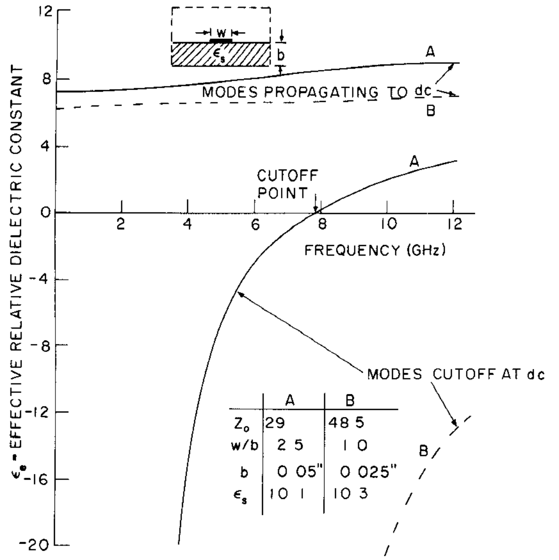

Fig. 3. DC propagating and cutoff modes in microstrip from coupled-line model.

$$\omega_0^2 = \frac{\mathcal{K}^2 v_0^2}{\bar{\epsilon}(1 - k^2)} \tag{11}$$

where ω_0 is the cutoff frequency of the mode which does not propagate at dc. This relation probably should be used with some caution since the parameters of the model are based solely on the functional mode, but (11) may be useful when considering the high-frequency limitations of microstrip.

Acknowledgment

The author wishes to thank Prof. P. Penfield, Jr., of the Massachusetts Institute of Technology, and Prof. P. McIsaac of Cornell University for helpful discussions.

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A Small-Signal and Noise Equivalent Circuit for IMPATT Diodes

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***Abstract*—A frequency-independent small-signal equivalent circuit for an IMPATT diode is proposed. It incorporates five circuit elements, including a negative resistance, and is valid over an octave range of frequency. With the addition of two white noise sources it also serves as a noise equivalent circuit.**

Introduction

An equivalent circuit of an electron device is a linear network having the same terminal properties as the device. Equivalent-circuit representations have been established for many electron devices because they facilitate the study of effects related to the external circuit. Frequency-independent equivalent circuits are particularly useful because they permit the use of simple circuit analysis techniques and aid in the study of the frequency variation of device performance.

For a nonlinear two-terminal negative-resistance device like an IMPATT diode, a linear equivalent circuit can be found for the small-signal (linearized) behavior of the device and for a limited frequency range of validity. The purpose of this short paper is to present a small-signal equivalent circuit and a noise equivalent circuit for IMPATT diodes. These equivalent circuits approximate only the terminal behavior of the diode;[1] no physical significance is attached to the circuit elements.

No frequency-independent lumped-noise equivalent circuit for IMPATT diodes has been reported so far. Haus *et al.* [1] have found a noise model for IMPATT diodes in the form of a transmission line with distributed noise sources which is not as convenient as a lumped-noise equivalent circuit. Johnson and Robinson [2] have, on the other hand, used a frequency-dependent model formed by separating the IMPATT-diode impedance into avalanche-region and drift-region impedances and connecting a noise source with the avalanche-region impedance.

A suitable small-signal equivalent circuit is also not available in the literature. The results of most theoretical calculations [3]–[6] and experimental measurements [7]–[9] of the small-signal impedance of IMPATT diodes have been expressed as a frequency-dependent admittance. Steinbrecher and Peterson [10] have proposed a frequency-independent small-signal model which is accurate only in the limit of low frequency ($\omega\tau_d \lesssim \pi/4$, where τ_d is the drift-region transit time) and predicts a diode negative conductance whose magnitude increases monotonically with frequency to an asymptotic value. Typical X-band diodes, however, have a maximum negative conductance at a frequency where $\omega\tau_d \approx 0.8\pi$, or higher for higher bias current [11], above which the magnitude of conductance decreases with increasing frequency; the model in [10] is, therefore, not suitable in the most useful frequency range of the diodes. Hulin *et al.*, [12] have also reported a circuit representation for the avalanche region of IMPATT diodes.

A frequency-independent small-signal equivalent circuit for an avalanche transit-time diode operated in the IMPATT mode, incorporating a negative resistance as the active element, is presented here. A noise equivalent circuit can also be derived from this small-signal model by incorporating two noise current sources in the circuit. Both sources are constant and frequency independent and are fully correlated with each other.

The equivalent circuit of the package in which the IMPATT diode is mounted is usually considered an integral part of the diode. In experimental evaluation and use of the diode equivalent circuit presented here, the diode package will have to be accounted for [7], [8]. An equivalent circuit for the package (which depends upon the method of mounting the package in a cavity) should, therefore, be added to the diode equivalent circuit.

Method of Determination

The two basic methods for determining the equivalent circuit for a given diode are the following.

From $Y_D(\omega)$ and $\overline{e_n^2}(\omega)$

For accurate modeling, the equivalent circuit is evaluated using the small-signal diode admittance $Y_D(\omega)$ and the mean-square open-circuit noise voltage per unit bandwidth $\overline{e_n^2}(\omega)$ at the diode terminals. For a given diode, these may be determined either directly by experimental measurement (and de-imbedding the diode from its circuit [10]) or indirectly, by first determining the diode structure (i.e., doping profile by CV measurement) and then carrying out theroetical calculations using a model such as the small-signal analysis of Gummel and Blue [6] which can be used for calculating both $Y_D(\omega)$ and $\overline{e_n^2}(\omega)$ numerically. In either case, the values of network elements in

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[1] A small-signal equivalent circuit of the device will be defined as one having approximately the same terminal impedance as the small-signal device impedance, and a noise equivalent circuit of the device as one for which both the impedance and the open-circuit noise voltage are close to those for the device. Obviously, a noise equivalent circuit will also serve as a small-signal equivalent circuit upon omitting the noise sources from it.